The role of cross-domain error correlations in strongly coupled 4D-Var atmosphere-ocean data assimilation

Amos S. Lawless Work with Polly J. Smith & Nancy K. Nichols University of Reading

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Strongly coupled incremental 4D-Var

- Control vector contains both atmosphere & ocean model variables
- Fully coupled tangent linear & adjoint models
- Allows for cross-domain covariances between atmosphere & ocean forecast errors

$$\mathbf{B}_{0} = \begin{pmatrix} \mathbf{B}_{\mathrm{AA}} & \mathbf{B}_{\mathrm{AO}} \\ \mathbf{B}_{\mathrm{AO}}^{\mathrm{T}} & \mathbf{B}_{\mathrm{OO}} \end{pmatrix}$$

- Atmosphere observations can influence ocean analysis and vice versa
- Leads to greater balance

first guess
$$\mathbf{x}_{0}^{(0)} = \mathbf{x}_{b}$$

non-linear trajectory computed using **coupled** model
 $\mathbf{x}_{i}^{(k)} = m(t_{i}, t_{0}, \mathbf{x}_{0}^{(k)})$
innovations $\mathbf{d}_{i}^{(k)} = \mathbf{y}_{i} - h(\mathbf{x}_{i}^{(k)})$
perturbation first guess $\delta \mathbf{x}_{i}^{(k)} = 0$
 $\mathbf{x}_{0}^{(k)}$
TL of **coupled** model: $J^{(k)}$
ADJ of **coupled** model: $\nabla J^{(k)}$
update $\mathbf{x}_{0}^{(k+1)} = \mathbf{x}_{0}^{(k)} + \delta \mathbf{x}_{0}^{(k)}$





Idealised system

single-column, coupled atmosphere-ocean model

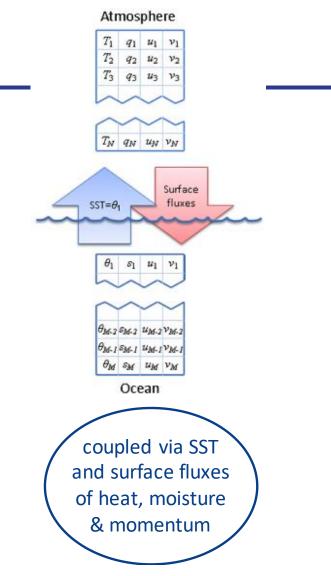
Atmosphere

- simplified version of the ECMWF single column model adiabatic component + vertical diffusion (no convection)
- 4 state variables on 60 model levels (surface to ~0.1hPa)
- forced by large scale horizontal advection

Ocean

- K-Profile Parameterisation (KPP) mixed-layer model
- 4 state variables on 35 model levels (1-250m)
- forced by short and long wave radiation at surface

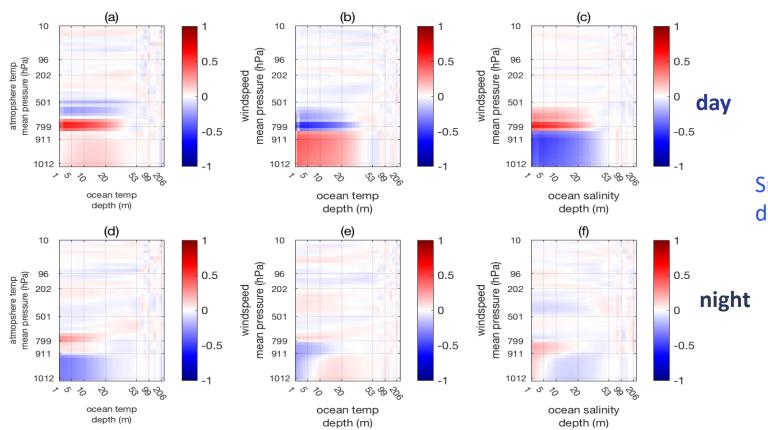
Smith et al 2015, *Tellus*, doi:10.3402/tellusa.v67.27025







December case



Smith et al 2017, MWR, doi:10.1175/MWR-D-16-0284.1

Atmosphere-ocean cross-correlation. Left to right: atmosphere-ocean temp, wind speed-ocean temp, wind speed-ocean salinity



National Centre for Earth Observation

Amos S. Lawless a.s.lawless@reading.ac.uk



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Method

- Idealised twin experiments, 12 hr window.
- Define

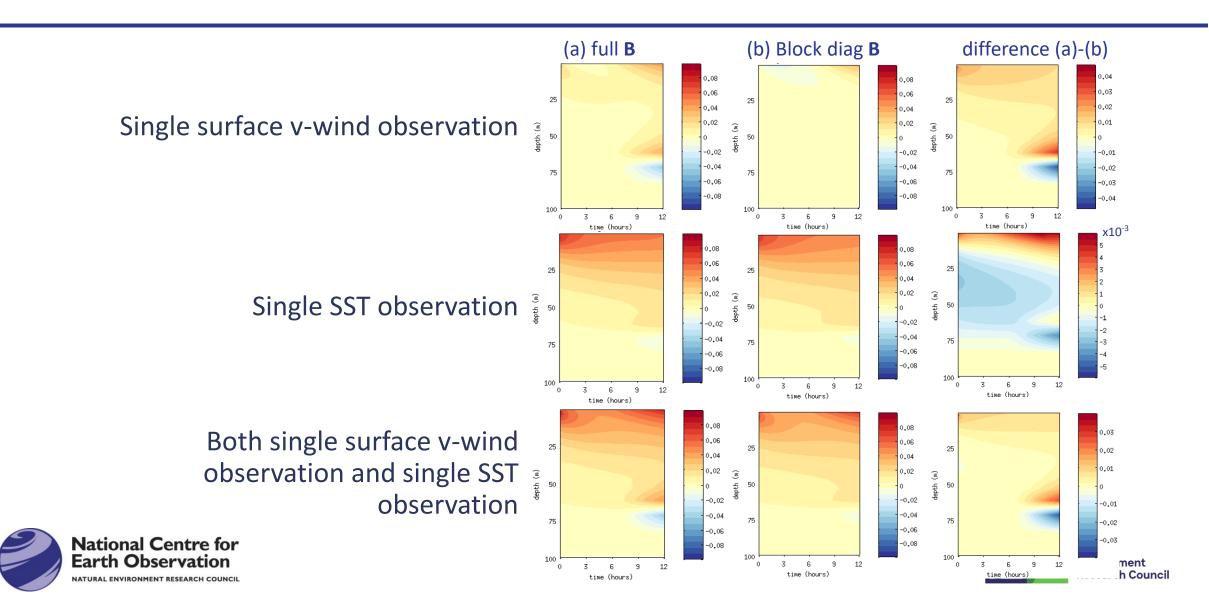
$$\mathbf{B}^{\mathsf{full}} = \begin{pmatrix} \mathbf{B}_{\mathsf{A}\mathsf{A}} & \mathbf{B}_{\mathsf{A}\mathsf{O}} \\ \mathbf{B}_{\mathsf{A}\mathsf{O}}^{\mathsf{T}} & \mathbf{B}_{\mathsf{O}\mathsf{O}} \end{pmatrix} \qquad \qquad \mathbf{B}^{\mathsf{diag}} = \begin{pmatrix} \mathbf{B}_{\mathsf{A}\mathsf{A}} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_{\mathsf{O}\mathsf{O}} \end{pmatrix}$$

- Use eigenvectors of correlation matrix for control variable transform.
- Compare assimilation observing only meridional wind *v*, only SST and both, at end of time window.
- Investigate effect of explicit and implicit covariances.

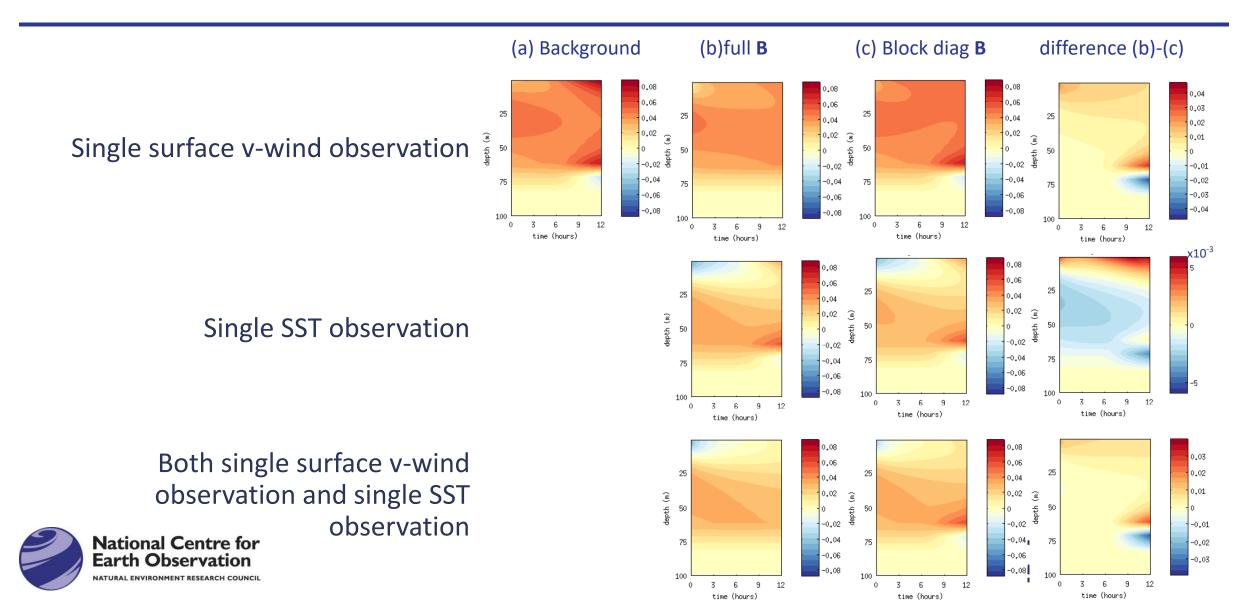




Analysis increments: Ocean temperature



Analysis errors: Ocean temperature



Conclusions

If only a single domain is observed:

- including explicit cross-domain forecast error covariances (B_{AO}≠ 0) mostly impacts the unobserved domain.
- if B_{AO} = 0 the initial increments in the unobserved domain rely on the implicitly generated cross-domain error covariances, which in turn depend on the strength of coupling in the TL model.
- setting B_{AO} = 0 will always lead to a loss of information; the unobserved domain is unable to influence the structure of the increments in the observed domain and so is unlikely to produce a balanced initial state.

If both domains observed:

• Explicitly specified covariances may still allow a better-balanced analysis state.



Smith, P.J., Lawless, A.S. and Nichols, N.K. (2020), *Q.J.R.M.S.* doi.org/10.1002/qj.3802.



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