Model and theory used for Kalman Filter: Earth Observation Example

Now we describe a slightly complex yet far from perfect model to predict the temperature changes that we're going to see over the concrete ground.

As the sun drives all but the tiniest temperature changes on Earth, it is logical to use this as the starting point for our model. First we calculate the amount of solar radiation arriving at the top of the Earth's atmosphere using the following equation,

$$S_{TOA} = S_0(\cos(H)\cos(\varphi)\cos(\delta) - \sin(\varphi)\sin(\delta))$$

S_{TOA} = Incoming radiation at the Top Of the Atmosphere, Wm^{-2}.
S_0 = Solar Constant, Wm^{-2}.
H = The observation matrix, [p x n].
φ = Solar declination, radians.
δ = Latitude of the observer, radians.

As the day progresses then the change in hour angle determines the increase and decrease in the calculated solar energy.

This radiation is transmitted down through the atmosphere and some is absorbed in the process, calculating this absorption of radiation is the next step in our model. Here, however, we make an enormous and sweeping assumption for the sake of keeping the model simple: we assume that the atmosphere is composed of a single layer and that this layer is uniform in temperature. Moreover, we define a simple constant to describe how much of the sun's energy is absorbed by the atmosphere:

$$S_{atmos}^{sun} = \alpha_{atmos}^{sun} S_{TOA}$$

We assume that all the remaining unabsorbed radiation passes unperturbed through the atmosphere and hits the ground. Here some of that radiation is absorbed by the ground and we define another constant to describe this absorption. In fact, if you look at the python code, you will see that in this case we have assumed that the surface is a perfect black body and so it absorbs all of the remaining radiation – it's not essential to do this, and you can change the numbers to see what difference it makes.

$$S_{ground}^{sun} = \alpha_{ground}^{sun} S_{TOA}$$

This is nearly enough on its own for a very simple model of ground and atmosphere temperature, except for the fact that it will lead to infinite heating. We must incorporate some emission of radiation from the atmosphere and ground (aka cooling) into our model. Here we use the Stephan Boltzmann law of radiative cooling to say that:

$$E_{ground} = \sigma(T_{ground})^4$$
$$E_{atmos} = \sigma(T_{atmos})^4$$

If you look carefully at these last two equations, you will see that the amount of radiation emitted by either the ground or the atmosphere is proportional to the fourth power of that medium's temperature. This is an important point because it immediately introduces complexity and feedback into our model – it makes the model non-linear. We'll discuss the implications of this later, but it's important to remember that fact.

To finish off the basics – some of this emitted radiation is then subsequently absorbed by the neighbouring medium (i.e. some of E (ground) is absorbed by the atmosphere) and so we have two final radiation terms to incorporate that information:

$$S^{ground}{}_{atmos} = \alpha^{ground}{}_{atmos} E_{ground}$$
$$S^{atmos}{}_{ground} = \alpha^{atmos}{}_{ground} E_{atmos}$$

Now that we know all the respective radiative absorptions and emissions of the ground and the atmosphere, we can combine these with the assumed values of mass and heat capacity for each medium in order to calculate the changes in temperature over time.

$$dT_{ground}/dt = (S^{sun}{}_{ground} + S^{atmos}{}_{ground} - E_{ground})/(C_{ground} M_{ground})$$

$$dT_{atmos}/dt = (S^{sun}{}_{atmos} + S^{ground}{}_{atmos} - E_{atmos})/(C_{atmos} M_{atmos})$$

Given that we ignore almost all aspects of the natural system – from humidity and clouds through to wind and atmospheric composition – and indeed large swathes of basic physics, this is clearly an incredibly simple model. Yet, it is the very imperfection of the model which allows us to showcase the power of the Kalman filter.

Now that we have our model and our observations, it's time to move on to the business of Kalman Filtering.

In this EO Kalman Filter example we're building on Greg Czeniak's undergraduate guide to Kalman filters. Aside from increasing the complexity a little in response to the non-linear model that we built on the previous page, we're also going to be modifying the terminology so that it's a little more in line with the kinds of terms you're likely to hear/read/use in Earth Observation circles.